

On Gauge Equivalence of Tachyon Solutions in Cubic Neveu-Schwarz String Field Theory

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April 29, 2010

Abstract

Simple analytic solution to cubic Neveu-Schwarz String Field Theory including the $GSO(-)$ sector is presented. This solution is an analog of the Erler-Schnabl solution for bosonic case and one of the authors solution for the pure $GSO(+)$ case. Gauge transformations of the new solution to others known solutions for the NS string tachyon condensation are constructed explicitly. This gauge equivalence manifestly supports the early observed fact that these solutions have the same value of the action density.

1 Introduction

The first nontrivial vacuum solution to open string field theory (SFT) equation of motion has been found in the seminal Schnabl paper [1]. This solution can be presented as a singular limit of a pure gauge solution configuration [1, 2]. Schnabl has found this solution in the Witten bosonic SFT [3]. The similar solution in the AMZ-PTY super SFT [4, 5] has been found by Erler [6]. The solution in fermionic SFT including the $GSO(-)$ sector [7] has been found in [8, 9]. The Erler solution Φ_E and our solution $\widehat{\Phi}_{AGM}$ both satisfy to the first and third Sen conjectures. Since both of them are singular limit of the pure gauge configurations it is tempting to expect that they are related via a gauge transformation [10].

To work with constructed vacuum solutions it would be nice to perform smoothening gauge transformations. The smooth form of the vacuum solution to bosonic SFT has been recently found by Schnabl and Erler [11]. Following their terminology we call this solution as a simple analytic one.

The simple analytic solution to cubic SSFT equation of motion is constructed in [12]. The goal of this paper is to present a simple analytic solution to fermionic SFT equation of motion with non-zero $GSO(-)$ sector. Like to the previous cases [11, 12] this new solution involves a continuous integral of wedge state and no singular limits are necessary. We will show the gauge equivalence of this solution to the simple pure $GSO(+)$ solution. It was natural to expect this result, since both solutions have the same value of action. However this result is not trivial since both solutions are not pure gauge.

We also present the gauge transformation which relates the new analytic solution to the AGM analytic solution [8]. Note that the gauge transformation that relates the pure $GSO(+)$ simple solution Φ_G and the corresponding solution with phantom terms [6] has been presented in [12].

We have the following picture

$$\begin{array}{ccc} \Phi_E & \xrightarrow{U_{E,G}} & \Phi_G \\ & & \\ \widehat{\Phi}_{AGM} & \xrightarrow{\widehat{U}_{AGM,new}} & \widehat{\Phi}_{new} \end{array}$$

The first line means that there is a gauge transformation, $U_{E,G}$, from Φ_E to Φ_G and the second line means that there is a transformation, $\widehat{U}_{AGM,new}$, from $\widehat{\Phi}_{AGM}$ to the solution presented in this paper.

As has been mentioned before, we can construct the gauge transformation, $\widehat{U}_{G,new}$, between two solutions Φ_G and $\widehat{\Phi}_{new}$. This means that we can close the diagram

$$\begin{array}{ccc}
\Phi_E & \xrightarrow{U_{E,G}} & \Phi_G \\
\downarrow \widehat{U}_{E,AGM} & & \downarrow \widehat{U}_{G,new} \\
\widehat{\Phi}_{AGM} & \xrightarrow{\widehat{U}_{AGM,new}} & \widehat{\Phi}_{new}
\end{array}$$

and

$$U_{E,G} \otimes I \cdot \widehat{U}_{G,new} = \widehat{U}_{E,AGM} \cdot \widehat{U}_{AGM,new} \quad (1)$$

Values of the action for the Erler solution Φ_E , the new simple solution Φ_G and the cubic NS SFT with $GSO(-)$ sector $\widehat{\Phi}_{AGM}$, were calculated in [6], [12, 14] and [8, 9], respectively and gave the same result

$$S[\Phi_E] = S[\Phi_G] = S[\widehat{\Phi}_{AGM}] = \frac{1}{2\pi^2}. \quad (2)$$

2 Notations

We use string fields K, B, c, γ in split string notation [6, 8, 13], these fields have the following BRST variations

$$\begin{aligned}
Qc &= cKc - \gamma^2, & Q\gamma &= cK\gamma - \frac{1}{2}\gamma Kc - \frac{1}{2}\gamma cK, \\
Q\gamma^2 &= cK\gamma^2 - \gamma^2 Kc, & QB &= K,
\end{aligned} \quad (3)$$

and satisfy the algebraic relations

$$\begin{aligned}
\{B, c\} &= 1, & [B, \gamma] &= 0, & [c, \gamma] &= 0, \\
[B, K] &= 0, & [K, c] &= \partial c, & [K, \gamma] &= \partial \gamma, \\
B^2 &= c^2 = 0.
\end{aligned} \quad (4)$$

Hereinafter we will use $(1+K)^{-1}$. We rewrite it using the Schwinger parameterization [11]

$$\frac{1}{1+K} = \int_0^\infty dt e^{-t(1+K)} = \int_0^\infty dt e^{-t\Omega^t}. \quad (5)$$

3 New solutions

Here we present a new solution to the fermionic string field equations of motion

$$\begin{aligned}
Q\Phi_+ + \Phi_+ \star \Phi_+ - \Phi_- \star \Phi_- &= 0, \\
Q\Phi_- + \Phi_+ \star \Phi_- - \Phi_- \star \Phi_+ &= 0.
\end{aligned} \quad (6)$$

The simple analytical solution is¹

$$\begin{aligned}\Phi_+ &= (c(1+K)Bc + 2B\gamma^2)\frac{1}{1+K}, \\ \Phi_- &= \left(\gamma + cBK\gamma + \frac{1}{2}\gamma KBc + \frac{1}{2}\gamma BcK\right)\frac{1}{1+K}.\end{aligned}\tag{8}$$

This solution is a generalization of the solution to the superstring equation of motion [12]

$$\Phi = (c(1+K)Bc + B\gamma^2)\frac{1}{1+K},\tag{9}$$

which in part is a generalization of Erler-Schnabl's solution to bosonic field equation of motion [11].

To derive the gauge equivalence (8) and (9) it is useful to introduce two functions [11]

$$f = 1, \quad g = \frac{1}{1+K},\tag{10}$$

and rewrite (8) and (9) in the form

$$\begin{aligned}\Phi_+ &= fc\frac{KB}{1-fg}cg + f\gamma\frac{KB}{1-fg}\gamma g + fB\gamma(1-K)\gamma g, \\ \Phi_- &= fc\frac{KB}{1-fg}\gamma g + f\gamma\frac{KB}{1-fg}cg - \frac{1}{2}f\gamma KBcg + \frac{1}{2}f\gamma BcKg,\end{aligned}\tag{11}$$

and

$$\Phi = fc\frac{KB}{1-fg}cg + fB\gamma^2g.\tag{12}$$

If we choose $f = g = F$ we get the AGM solution and the Erler solution.

4 Gauge equivalence of solutions

4.1 $U_{E,G}$ and $\widehat{U}_{AGM,new}$

Following [11] one can built a gauge transformation between (12) and Erler's solution Φ_E [12]

$$\Phi_E = U_{E,G}^{-1}(\Phi_G + Q)U_{E,G},\tag{13}$$

¹We can also construct the following ill-defined solution

$$\begin{aligned}\Phi_+ &= (c + B\gamma^2)(1 - K) + B\gamma^2, \\ \Phi_- &= \gamma - \frac{1}{2}\gamma BcK + \frac{1}{2}\gamma KBc,\end{aligned}$$

which formally satisfies the equation of motion (6). In the case of the pure $GSO(+)$ sector the ill-defined solution has the form [14]

$$\Phi_+ = (c + B\gamma^2)(1 - K).\tag{7}$$

where

$$\begin{aligned} U_{E,G} &= 1 - fBcg + Mf'Bcg', \\ U_{E,G}^{-1} &= 1 - f'Bcg' + M^{-1}fBcg, \end{aligned} \quad (14)$$

and the function M is defined as

$$M = \left(\frac{1 - fg}{1 - f'g'} \right), \quad (15)$$

here $f' = g' = F$ and f, g are defined by (10).

The gauge transformation between (11) and the AGM solution $\hat{\Phi}_{AGM}$ is

$$\hat{\Phi}_{AGM} = \hat{U}_{AGM,new}^{-1} (\hat{\Phi}_{new} + \hat{Q}) \hat{U}_{AGM,new} \quad (16)$$

where

$$\begin{aligned} \hat{U}_{AGM,new} &= U_+ \otimes I + U_- \otimes \sigma_1, \\ \hat{U}_{AGM,new}^{-1} &= U_+^{-1} \otimes I + U_-^{-1} \otimes \sigma_1, \end{aligned} \quad (17)$$

and

$$\begin{aligned} U_+ &= (1 - fBcg + Mf'Bcg'), & U_- &= (-fB\gamma g + Mf'B\gamma g'), \\ U_+^{-1} &= (1 - f'Bcg' + M^{-1}fBcg), & U_-^{-1} &= (-f'B\gamma g' + M^{-1}fB\gamma g). \end{aligned} \quad (18)$$

Also we denote

$$\hat{Q} = Q \otimes \sigma_3, \quad \hat{\Phi} = \Phi_+ \otimes \sigma_3 + \Phi_- \otimes i\sigma_2. \quad (19)$$

4.2 $\hat{U}_{G,new}$

Here we build the gauge transformation connecting two solutions: solution to super and fermionic field theory equation of motion.

Let us consider the gauge transformation

$$\hat{\Phi}' = \hat{U}^{-1} (\hat{\Phi} + \hat{Q}) \hat{U} \quad (20)$$

and rewrite it in the components

$$\begin{aligned} \Phi'_+ &= U_+^{-1} (\Phi_+ + Q) U_+ - U_-^{-1} (\Phi_+ + Q) U_- + U_+^{-1} \Phi_- U_- - U_-^{-1} \Phi_- U_+, \\ \Phi'_- &= U_+^{-1} (\Phi_+ + Q) U_- - U_-^{-1} (\Phi_+ + Q) U_+ + U_+^{-1} \Phi_- U_+ - U_-^{-1} \Phi_- U_-. \end{aligned} \quad (21)$$

We take Φ_+ and Φ_- in the form (11) and take $U_+ = U_+^{-1} = I$ and $U_- = -fB\gamma g$, $U_-^{-1} = fB\gamma g$.

Then the first line of (21) gives

$$\begin{aligned}
U_+^{-1}(\Phi_+ + Q)U_+ &= fc \frac{KB}{1-fg}cg + f\gamma \frac{KB}{1-fg}\gamma g + fB\gamma(1-K)\gamma g, \\
U_-^{-1}(\Phi_+ + Q)U_- &= -f\gamma \frac{KB}{1-fg}f^2g^2\gamma g - f\gamma KBfg\gamma g, \\
U_+^{-1}\Phi_-U_- &= -f\gamma \frac{KB}{1-fg}fg\gamma g, \\
U_-^{-1}\Phi_-U_+ &= f\gamma \frac{KB}{1-fg}fg\gamma g.
\end{aligned} \tag{22}$$

So we have the following expression for Φ'_+

$$\Phi'_+ = fc \frac{KB}{1-fg}cg + fB\gamma^2g. \tag{23}$$

For the second line of (21) we have

$$\begin{aligned}
U_+^{-1}(\Phi_+ + Q)U_- &= -fc \frac{KB}{1-fg}fg\gamma g - fcKB\gamma g - \frac{1}{2}f\gamma KBcg - \frac{1}{2}f\gamma BcKg, \\
U_-^{-1}\Phi_+U_+ &= f\gamma \frac{KB}{1-fg}fgcg, \\
U_+^{-1}\Phi_-U_+ &= fc \frac{KB}{1-fg}\gamma g + f\gamma \frac{KB}{1-fg}cg - \frac{1}{2}f\gamma KBcg + \frac{1}{2}f\gamma BcKg, \\
U_-^{-1}\Phi_-U_- &= 0,
\end{aligned} \tag{24}$$

thereby, for Φ'_- we have

$$\Phi'_- = 0. \tag{25}$$

Thus, we have the gauge equivalence of two solutions.

5 Conclusion

The construction presented in this paper permits to conclude that all the tachyon condensation solutions are gauge equivalent. In standard Siegel gauge the tachyon leaves in the $GSO(-)$ sector and breaks the supersymmetry of the model. It is also can be expected that the analytical solution [6] leaving in the pure $GSO(+)$ in spirit of year considerations in [15] breaks the supersymmetry. To check this statement one has to also deal with the R-sector. It would be interesting to incorporate the Kroyter idea of constructing the NSR SFT whose string fields carry an arbitrary picture number and reside in the large Hilbert space [16].

Acknowledgements

The work is supported in part by RFBR grant 08-01-00798, NS-795.2008.1 and FCPK-02.740.11.5057.

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